Number Patterns

1. The sequence below can be formed by adding.

$$0 + 2 = 2 \times 1$$
$$0 + 2 + 4 = 3 \times 2$$
$$0 + 2 + 4 + 6 = 4 \times 3$$
$$0 + 2 + 4 + 6 + 8 = 5 \times 4$$

(a) Express $0 + 2 + 4 \ldots\ldots + 16 + 18$ in the same way.
(b) The first **n** terms of this sequence are added. Write down an expression, in **n**, for the total.

2. By adding terms the following sequence can be obtained.

Number of terms	
2	$1 + 9 = 2(4 \times 2 - 3)$
3	$1 + 9 + 17 = 3(4 \times 3 - 3)$
4	$1 + 9 + 17 + 25 = 4(4 \times 4 - 3)$
5	$1 + 9 + 17 + 25 + 33 = 5(4 \times 5 - 3)$

(a) Write down the sum of the first 10 terms in the same way.
(b) The first **n** terms of the sequence are added. Write down an expression for the total.

3. The sequence of numbers starting at 3 and adding 4 is

3, 7, 11, 15, 19, …………..

Consecutive numbers of this sequence can be added to give

$$3 + 7 = 2 \times 2^2 + 2$$
$$3 + 7 + 11 = 2 \times 3^2 + 3$$
$$3 + 7 + 11 + 15 = 2 \times 4^2 + 4$$
$$3 + 7 + 11 + 15 + 19 = 2 \times 5^2 + 5$$

(a) Express $3 + 7 + 11 + \ldots\ldots..31 + 35$ in the same way.
(b) Write down an expression for the first **n** terms of this sequence.

4. The sequence below is formed by adding consecutive squares.

$$1^2 + 2^2 = 2(2^2 - 2) + 1$$
$$2^2 + 3^2 = 2(3^2 - 3) + 1$$
$$3^2 + 4^2 = 2(4^2 - 4) + 1$$
$$4^2 + 5^2 = 2(5^2 - 5) + 1$$

(a) Write $19^2 + 20^2$ in the same way.
(b) Write down an expression for $\mathbf{(n - 1)^2 + n^2}$.

5. The following pattern can be obtained by adding.

Number of terms	
2	$5 + 11 = 2(3 \times 2 + 2)$
3	$5 + 11 + 17 = 3(3 \times 3 + 2)$
4	$5 + 11 + 17 + 23 = 4(3 \times 4 + 2)$

(a) Write down an expression for the sum of the first 20 terms and use your expression to find this sum.
(b) Write down an expression for the sum of the first **n** terms.

6. The sum of the squares of consecutive even numbers can be written

$$2^2 + 4^2 = (2 + 4)^2 - 2 \times 2 \times 4$$
$$4^2 + 6^2 = (4 + 6)^2 - 2 \times 4 \times 6$$
$$6^2 + 8^2 = (6 + 8)^2 - 2 \times 6 \times 8$$

(a) Write, in the same way, an expression for $18^2 + 20^2$.
(b) Write down an expression for $\mathbf{n^2 + (n + 2)^2}$.

7. The difference between consecutive cubic numbers can be written

$$1^3 - 0^3 = 3 \times 1 \times 0 + 1$$
$$2^3 - 1^3 = 3 \times 2 \times 1 + 1$$
$$3^3 - 2^3 = 3 \times 3 \times 2 + 1$$
$$4^3 - 3^3 = 3 \times 4 \times 3 + 1$$

(a) In the same way, write down an expression for $10^3 - 9^3$
(b) Write down an expression for $\mathbf{n^3 - (n - 1)^3}$. Simplify this expression.

8. The sequence of odd numbers is 1,3,5,7,……

A pattern for the odd numbers can be written as follows

The **first** odd number: $1 = 1^2 - 0^2$
The **second** odd number: $3 = 2^2 - 1^2$
The **third** odd number: $5 = 3^2 - 2^2$

(a) Express the **fourth** odd number in the same way.
(b) Express 19 in the same way.
(c) Write down a formula for the **n**th odd number. **Simplify this expression.**

9. The sequence below is formed by adding consecutive terms.

Number of terms	
2	$1 + 11 = 2(5 \times 2 - 4)$
3	$1 + 11 + 21 = 3(5 \times 3 - 4)$
4	$1 + 11 + 21 + 31 = 4(5 \times 4 - 4)$

(a) Write down an expression for the sum of the first 12 terms of this sequence.
(b) The first **n** terms are added. Write down an expression for this sum.

10. The pattern below is formed by adding consecutive terms.

$$2 + 8 = 3 \times 2^2 - 2$$
$$2 + 8 + 14 = 3 \times 3^2 - 3$$
$$2 + 8 + 14 + 20 = 3 \times 4^2 - 4$$

(a) Write down the sum of the first 12 terms in the same way and calculate its value.
(b) Write down an expression for the sum of the first **n** terms of this sequence.

11. The sequence of numbers obtained by starting at 1 and adding 3 is

1, 4, 7 , 10, …………..

Terms of this sequence are added as follows

$$1 + 4 = \frac{2(3 \times 2 - 1)}{2}$$

$$1 + 4 + 7 = \frac{3(3 \times 3 - 1)}{2}$$

$$1 + 4 + 7 + 10 = \frac{4(3 \times 4 - 1)}{2}$$

(a) In the same way, write down an expression for 1 + 4 + 7 + ……….+ 25
(b) The first **n** terms of this sequence are added. Write down an expression for this sum.

12. The numbers 3, 10 , 21, 36, 55, ……
can be written as (1 x 3), (2 x 5), (3 x 7), (4 x 9), (5 x 11), ……

The sum of these numbers can be found using the following

2 terms: $(1 \times 3) + (2 \times 5) = \frac{2(2 + 1)(4 \times 2 + 5)}{6}$

3 terms: $(1 \times 3) + (2 \times 5) + (3 \times 7) = \frac{3(3 + 1)(4 \times 3 + 5)}{6}$

4 terms: $(1 \times 3) + (2 \times 5) + (3 \times 7) + (4 \times 9) = \frac{4(4 + 1)(4 \times 4 + 5)}{6}$

(a) Express the sum of the first 5 terms in the same way.

(b) Express the sum of the first **n** terms in the same way.

13. The sums of a particular sequence are given below.

Number of terms		
2	$1 + 1.5$	$= \dfrac{2^2 + 3 \times 2}{4}$
3	$1 + 1.5 + 2$	$= \dfrac{3^2 + 3 \times 3}{4}$
4	$1 + 1.5 + 2 + 2.5$	$= \dfrac{4^2 + 3 \times 4}{4}$

(a) Write down an expression for the sum of the first 10 terms of this sequence.
(b) Write down an expression for the sum of the first **n** terms of this sequence.

14. The following pattern can be used to sum consecutive square whole numbers.

$$1^2 + 2^2 = \frac{2 \times 3 \times 5}{6}$$

$$1^2 + 2^2 + 3^2 = \frac{3 \times 4 \times 7}{6}$$

$$1^2 + 2^2 + 3^2 + 4^2 = \frac{4 \times 5 \times 9}{6}$$

(a) Express $1^2 + 2^2 + 3^2 + \ldots\ldots.10^2$ in the same way.
(b) Express $1^2 + 2^2 + 3^2 + \ldots\ldots.n^2$ in the same way.

15. A number pattern is given below

1st term: $3^2 - 1^2$
2nd term: $4^2 - 2^2$
3rd term: $5^2 - 3^2$

(a) Write down a similar expression for the 4th term.
(b) Hence find the nth term in its simplest form.

16. Using the sequence

$$1, 3, 5, 7, 9, \ldots\ldots\ldots.$$

(a) Find S_3, the sum of the first 3 numbers.
(b) Find S_n, the sum of the first n numbers.
(c) Hence, or otherwise, find the $(n + 1)^{th}$ term of the sequence.

17. A number pattern is shown below.

$$1^3 + 1 = (1 + 1)(1^2 – 1 + 1)$$
$$2^3 + 1 = (2 + 1)(2^2 – 2 + 1)$$
$$3^3 + 1 = (3 + 1)(3^2 – 3 + 1)$$

(a) Write down a similar expression for $7^3 + 1$.
(b) Hence write down an expression for $n^3 + 1$.
(c) Hence find an expression for $8p^3 + 1$.

18. A sequence of numbers can be obtained using the process below.

Starting Numbers: 3, 4, 1

Row 1: **3** **4** **1**

Row 2: **3** **3 + 4** **4 + 1** **1**

Row 3: **3** **3 + 7** **7 + 5** **5 + 1** **1**

Total for row 3 = 32 = 4 x (3 + 4 + 1)

(a) Starting with the numbers -1, 7 and -3, show that the total for row 3 is still equal to 4 times the sum of the starting numbers.
(b) Starting with the values a, b and c, prove that the total for row 3 is **always** equal to 4 times the sum of the starting values.

19. The difference between the cubes of consecutive odd numbers form the pattern below.

$$3^3 – 1^3 = 2(3 + 1)^2 – 2 \times 3 \times 1$$
$$5^3 – 3^3 = 2(5 + 3)^2 – 2 \times 5 \times 3$$
$$7^3 – 5^3 = 2(7 + 5)^2 – 2 \times 7 \times 5$$
$$9^3 – 7^3 = 2(9 + 7)^2 – 2 \times 9 \times 7$$

(a) Write $15^3 – 13^3$ in the same way.
(b) Write down a similar expression for $n^3 – (n – 2)^3$.

(c) Given that $14^3 – 10^3$ can be written as $16(7 + 5)^2 – 16 \times 7 \times 5$, find an expression for $21^3 – 15^3$.

20. The differences between the cubes of consecutive even numbers form the pattern below.

$$4^3 - 2^3 = 6 \text{ x } 4 \text{ x } 2 + 8$$
$$6^3 - 4^3 = 6 \text{ x } 6 \text{ x } 4 + 8$$
$$8^3 - 6^3 = 6 \text{ x } 8 \text{ x } 6 + 8$$

(a) Write 12^3- 10^3 in the same way.
(b) Write down a similar expression for $n^3 - (n - 2)^3$.
(c) The difference between the cubes of two consecutive even numbers is 2648. Use your answer to (b) to find these two numbers.

21. 7, -2, 5, 3, 8

In the sequence above, each term after the first two terms is the sum of the previous two terms.
For example: the third term is 5 = 7 + -2

(a) A sequence follows the above rule.
The first term is x and the second term is y. The 5th term is 5.

x, y, —, —, 5

Show that 2x + 3y = 5

(b) This is repeated for another sequence, first term y and second term x, 6th term 17.

y, x, —, —, —, 17

Write down another equation in x and y.

(c) Find the values of x and y.