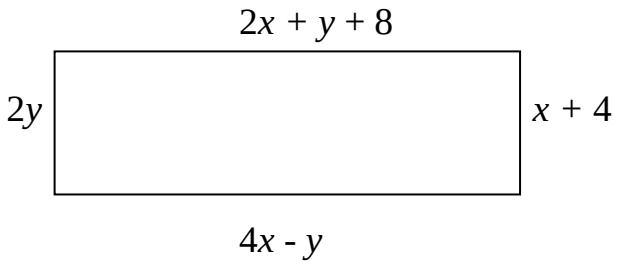
Applications of Simultaneous Equations. KS4 Higher.

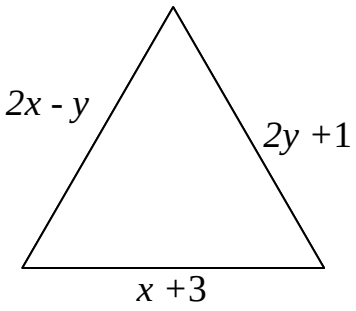
Calculators may be used.

NOTE: ALL DIAGRAMS ARE NOT DRAWN TO SCALE.

1. The lengths of the rectangle below are all in centimetres.
   (a) Work out the value of x and the value of y.
   (b) Hence, workout the area and perimeter of the rectangle.

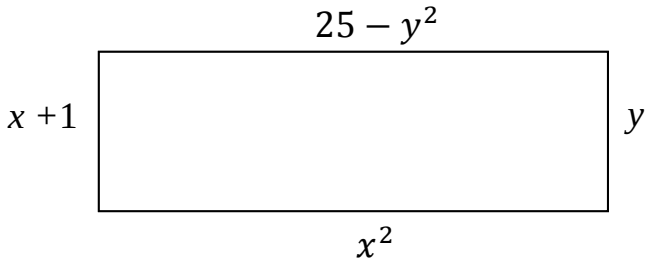


2. The lengths of the equilateral triangle below are given in centimetres.
   (a) Work out the value of x and the value of y.
   (b) Hence, workout the perimeter of the equilateral triangle.
   (c) Hence, workout the area of the equilateral triangle, giving your answer correct to 3sf.

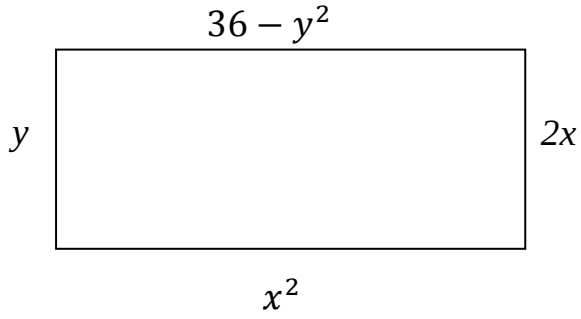


Simultaneous equations, one linear and one quadratic.

3. The lengths of the rectangle below are all in centimetres.
   Work out the value of *x* and *y*.

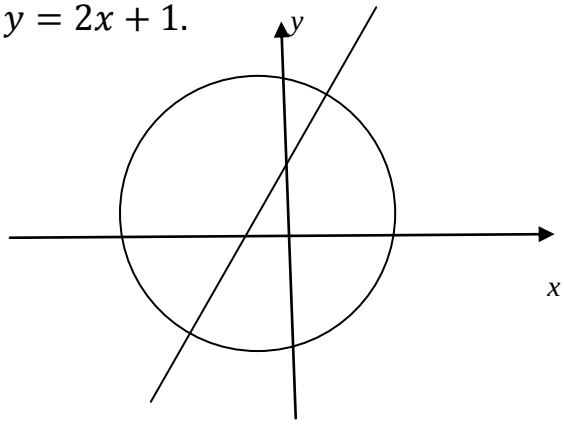


4. The lengths of the rectangle below are all in centimetres.
   Work out the value of *x* and *y*, giving your answer correct to 1dp.



5. Prove algebraically that the straight line
   $y = 5$ does not intersect the circle $x^2 + y^2 = 16$.

6. Work out the coordinates of the points of intersection of the circle
   $x^2 + y^2 = 34$ and the straight line $y = 2x + 1$.



Answers/Solutions (Solutions not unique).

(1) (a) Opposite sides are equal. $\Rightarrow$

$x + 4 = 2y$ — (1)

$4x - y = 2x + y + 8$ — (2)

from (2) $4x - 2x - 8 = 2y$

$2x - 8 = 2y$ — (3)

(1) = (3) $\Rightarrow$

$2x - 8 = x + 4$

$x = 12$

hence from (1) $12 + 4 = 2y$

$16 = y$

$8 = y$

Hence $x = 12, y = 8$

(b) $L = 4x - y = 4 \times 12 - 8 = 48 - 8 = 40$

$W = x + 4 = 12 + 4 = 16$

Area $= 40 \times 16 = 640\ cm^2$

Perimeter $= 2(L + W) = 2(40 + 16)$

$= 2(56) = 112$ cm

(2) (a) Lengths are equal. $\Rightarrow$

$x + 3 = 2y + 1$ — (1) and $x + 3 = 2x - y$

$\Rightarrow x = 2y - 2$ — (3) $y + 3 = x$ — (4)

(3) = (4) $2y - 2 = y + 3$

$y = 5$

from (4) $x = y + 3 = 5 + 3 = 8$

$x = 8, y = 5$

(b) Length of 1 side $= x + 3 = 8 + 3 = 11$.

Perimeter $= 3 \times 11 = 33$ cm

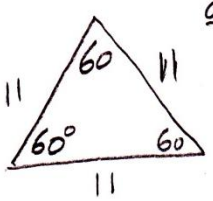
(2) Continued:

(c) "$A = \frac{1}{2}ab\sin C$"

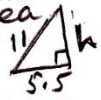
$A = \frac{1}{2} \times 11 \times 11 \times \sin 60$

$A = 52.394$

$= 52.4\text{cm}^2$ to 3sf



OR use Pythagoras To find the height and hence area



$h = 9.53$

(3) $y = x + 1$ —① opposite sides equal.

$x^2 = 25 - y^2$ —②

$\Rightarrow x^2 + y^2 = 25$

Substitute for y into ②

$x^2 + (x+1)^2 = 25$

$x^2 + x^2 + 2x + 1 = 25$

$2x^2 + 2x - 24 = 0$

$\div 2 \Rightarrow x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4$, $x = 3 \Rightarrow$ from ① $y = 3+1 = 4$

Void as $x+1 = -4+1 = -3$ would give a negative length!!

negative value of y also void.

$x = 3, y = 4$ Ans.

(4) $y = 2x$ and $x^2 = 36 - y^2$ opposite sides equal.

$x^2 + y^2 = 36$

$x^2 + (2x)^2 = 36$

$x^2 + 4x^2 = 36$

$5x^2 = 36$

$x^2 = 7.2$

$x = \pm 2.683$

$x = \pm 2.7$ to 1dp

$x = 2.7$, $y = 2 \times 2.7 = 5.4$ Ans.

negative values are void as lengths cannot be negative.

(5) Substitute $y=5$ into

$x^2+y^2=16$

$x^2+5^2=16$

$x^2+25=16$

$x^2=16-25=-9$

$x=\sqrt{-9}$ (no real values)

hence the line does not meet the circle.

(6) Substitute $y=2x+1$ into

$x^2+y^2=34 \Rightarrow$

$x^2+(2x+1)^2=34$

$x^2+4x^2+4x+1=34$

$5x^2+4x-33=0$

$(5x+11)(x+3)=0$

$x=\frac{11}{5}$ OR $x=-3$

$x=2.2$

hence $y=2\times2.2+1$

$y=4.4+1$

$y=5.4$

hence $y=2x-3+1$

$y=-6+1=-5$

(2.2, 5.4) and (-3, -5)

I hope you find this useful. If you find any errors, please let me know. Thank you.